# Mean Multifractal Properties of Low-Frequency Microseismic Noise

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## Introduction

Low-frequency microseismic field is an important source of geophysical information [Kobayashi & Nishida, 1998; Tanimoto et al. 1998; Tanimoto, Um, 1999; Tanimoto, 2001, 2005; Kurrle, Widmer-Schnidrig, 2006]. Atmospheric and oceanic processes are the main generators of low-frequency microseisms. But the energy from these sources is transferred through the Earth's crust. Thus, the changes within Earth's crust must influence the structure of noise and parameters of microseismic noise structure could reflect thin peculiarities of tectonic processes. Usual spectral analysis turns to be rather rough instrument which does not allow deep insight in the noise studies because low-frequency microsesimic noise does not include narrow-banded or monochromatic components. Investigation of fractal and multifractal properties of geophysical processes [Currenti et al., 2005; Ramirez-Rojas et al., 2005; Ida et al., 2005; Telesca et al., 2005] seems to be much more perspective in noise studies just because this approach allow explore signals which are not interesting from spectral point of view. This idea was the main in investigations [Lyubushin & Sobolev, 2006; Sobelev & Lyubushin, 2007; Lyubushin, 2007] of the noise multifractal structure and seeking precursors of strong earthquakes as coherent effects between variations of multifractal singularity spectra parameters calculated within adjacent "small" time intervals for different broadband seismic stations. In this article these investigation are continued in the direction of estimating a mean parameters of singularity spectra by averaging their values over a large number of broadband seismic stations, covering some seismically active region.

## Materials and Methods

For the analysis a vertical broadband seismic components with 1-second sampling (LHZ-records) from 54 F-net seismic stations from Japan were downloaded from internet address http://www.fnet.bosai.go.jp for time interval 1997 – April 2008. The whole list of F-net seismic stations includes 83 positions but a lot of them form a very dense spatial cluster especially in Central Japan. That is why only 54 stations were taken for analysis in order to avoid dominating information from some regions. The positions of these stations are presented on the Fig.1 with epicentre of Hokkaido earthquake 25.09.2003, M=8.3 which turns to be an important change point for behaviour of noise multifractal singularity spectra parameters.

Multifractal singularity spectrum $F(\alpha)$ of the signal $X(t)$ is defined as a fractal dimensionality of time moments $\tau_\alpha$ which have the same value of local Lipschitz-Holder exponent $h(t) = \lim_{\delta \to 0} \frac{\ln(\mu(t,\delta))}{\ln(2\delta)}$, i.e. $h(\tau_\alpha) = \alpha$, where $\mu(t,\delta) = \max_{t-\delta \le s \le t+\delta} X(s) - \min_{t-\delta \le s \le t+\delta} X(s)$ is a measure of signal variability in the vicinity of time moment $t$ [Feder, 1989]. If the signal $X(t)$ is a usual self-similar monofractal signal with

Hurst exponent value $0 < H < 1$ [Taqqu, 1988], then $F(H) = 1, \quad F(\alpha) = 0 \; \forall \alpha \neq H$ but finite sample estimate of singularity spectrum does not obey these rigorous theoretical conditions of course. Practically the most convenient method for estimating singularity spectrum is a DFA-method – detrended fluctuation analysis [Kantelhardt et al., 2002] which is used here. Technical details of computing could be found in [Lyubushin & Sobolev, 2006; Lyubushin, 2007].

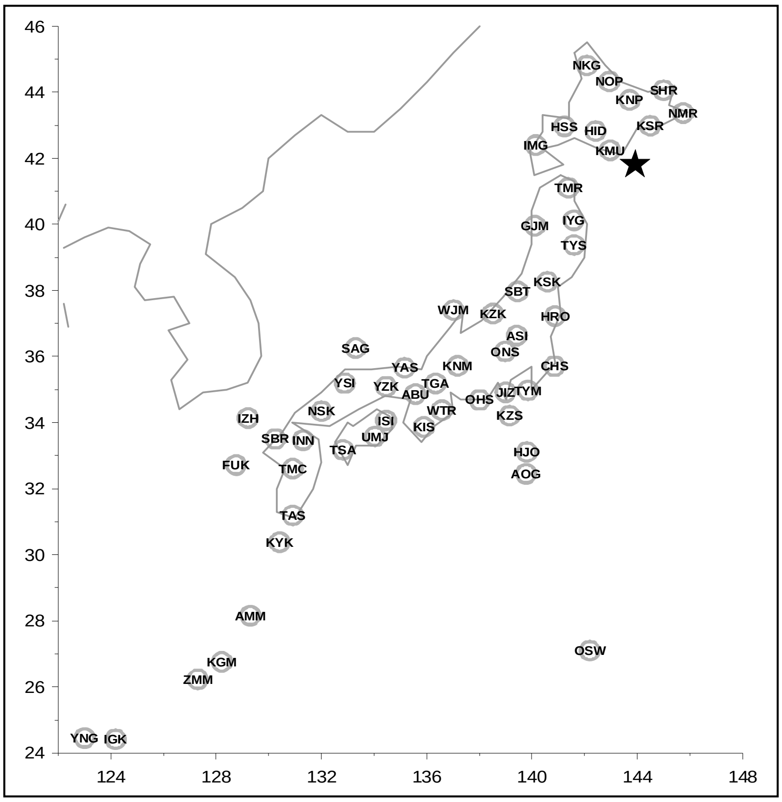


Figure 1. Circles – positions of 54 F-net broadband seismic stations in Japan with their 3-letters codes. Star – epicenter of Hokkaido earthquake, M = 8.3, 25.09.2003, Latitude = 41.81°, Longitude = 143.91°

Singularity spectra were estimated within adjacent time intervals of 0.5 hour length (1800 1-seconds samples) for 54 stations presented at the Fig.1. A typical graphics of singularity spectra estimate for one of the 0.5 hour time interval and for one of the stations is presented at the Fig.2. The function $F(\alpha)$ could be characterized by following parameters: $\alpha_{\min}, \alpha_{\max}, \Delta\alpha = \alpha_{\max} - \alpha_{\min}$ and $\alpha^*$ - an argument providing maximum to singularity spectra: $F(\alpha^*) = \max\limits_{\alpha} F(\alpha)$. Parameter $\alpha^*$ could be called a generalized Hurst exponent and it gives the most typical value of Lipschitz-Holder exponent.

Parameter $\Delta\alpha$ could be regarded as a measure of variety of stochastic behavior. It should be noticed that usually $F(\alpha^*) = 1$ – maximum of singularity spectra equals to the dimensionality of embedding set, i.e. to dimensionality of time interval. But time intervals could occur for which $F(\alpha^*) < 1$. It means that the behavior of the noise within these time intervals strongly differ from behavior of multifractal signal.

Computing median values of $\Delta\alpha$ and $\alpha^*$ over all seismic stations which have registration within current "small" 0.5 hours time interval produces an averaged time series of $(\Delta\alpha, \alpha^*)$-

variations which gather information from all station of the seismic net. The behavior of these time series is a main object for data analysis. Besides that another statistics were constructed similarly by calculating median values of $(\Delta\alpha, \alpha^*)$-variations for seismic records after coming to 1-minutes sampling time interval within "big" adjacent time intervals of the 1 day length (1440 1-minutes samples). These time series (more long periodic) were investigated as well. For estimating singularity spectra within 0.5 hour intervals local scale-depended trends were removed by polynomials of 4th order, whereas for the case of 1-day time interval – by polynomials of 8th order.

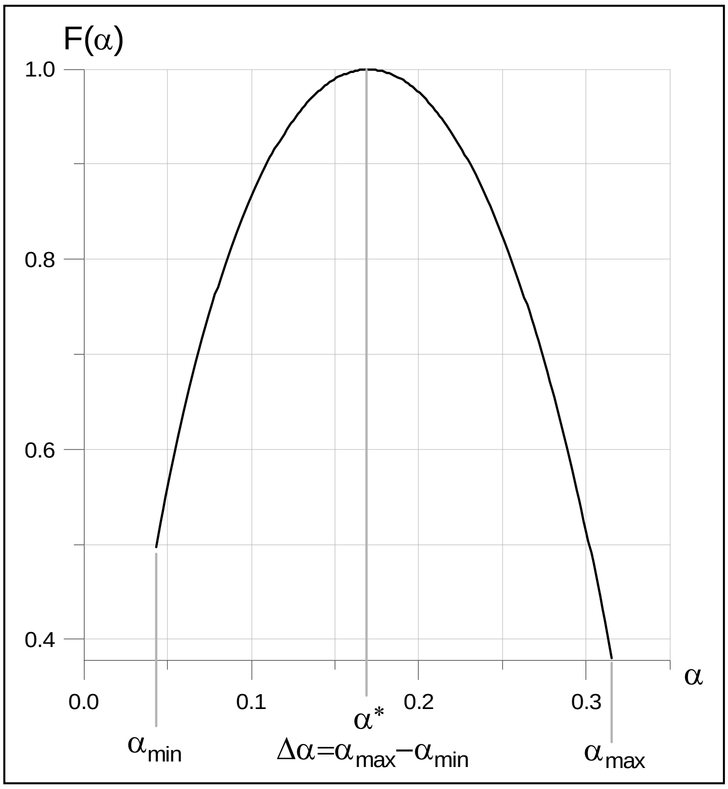


Figure 2. Example of multifractal singularity spectra estimate for 30-minutes time interval.

## Results and Discussion

Figures 3 and 4 present the main results. Figures 3(a) and 4(a) are identical and illustrate the sequence of strong earthquakes (M ≥ 6.0) inside a rather wide neighbourhood of Japan islands – in the rectangular 20° ≤ Latitude ≤ 60°, 120° ≤ Longitude ≤ 160°. The arrow indicates a time moment of Hokkaido earthquake 25.09.2003, M=8.3.

Figure 3(b) presents variations of median value of singularity spectra parameter $\Delta\alpha$, estimated within 0.5 hours time intervals (grey line) and result of its averaging using Gaussian kernel smoothing with averaging radius 200 days (bold black line). This Gaussian kernel trend $\bar{z}(t\,|\,r)$ with averaging radius $r > 0$ for the signal $z(t)$ is defined by the formula [Hardle, 1989]:

$$\bar{z}(t\,|\,r) = \int_{-\infty}^{+\infty} z(t + r\cdot\xi)\cdot\psi(\xi)\,d\xi \Bigg/ \int_{-\infty}^{+\infty} \psi(\xi)\,d\xi\,, \quad \psi(\xi) = \exp(-\xi^2)$$

The main peculiarity of the Fig.3(b) is a statistically significant change of $\Delta\alpha$ mean value from 0.322 for 1997-2003 till 0.307 for 2004-2008 which began 0.5 years before Hokkaido earthquake M=8.3, 25.09.2003. Taking into account interpretation of the parameter $\Delta\alpha$ as a measure of noise behavior variability, this effect could be regarded as a decreasing of hidden “number of freedom” of the Earth’s crust before the strong earthquake which dramatically changes the state of lithosphere as a transfer element from atmosphere-ocean processes to low-frequency microsesimic oscillations.

It should be noticed that using median values of singularity spectra parameters (like any other way of averaging) is some kind of extracting common signal which exists in variations of noise multifractal parameters at different stations but could be hidden within individual statistical fluctuations of finite sample singularity spectra estimates. Besides that, using median is a robust way of struggle with existence of gaps in registration at different seismic stations.

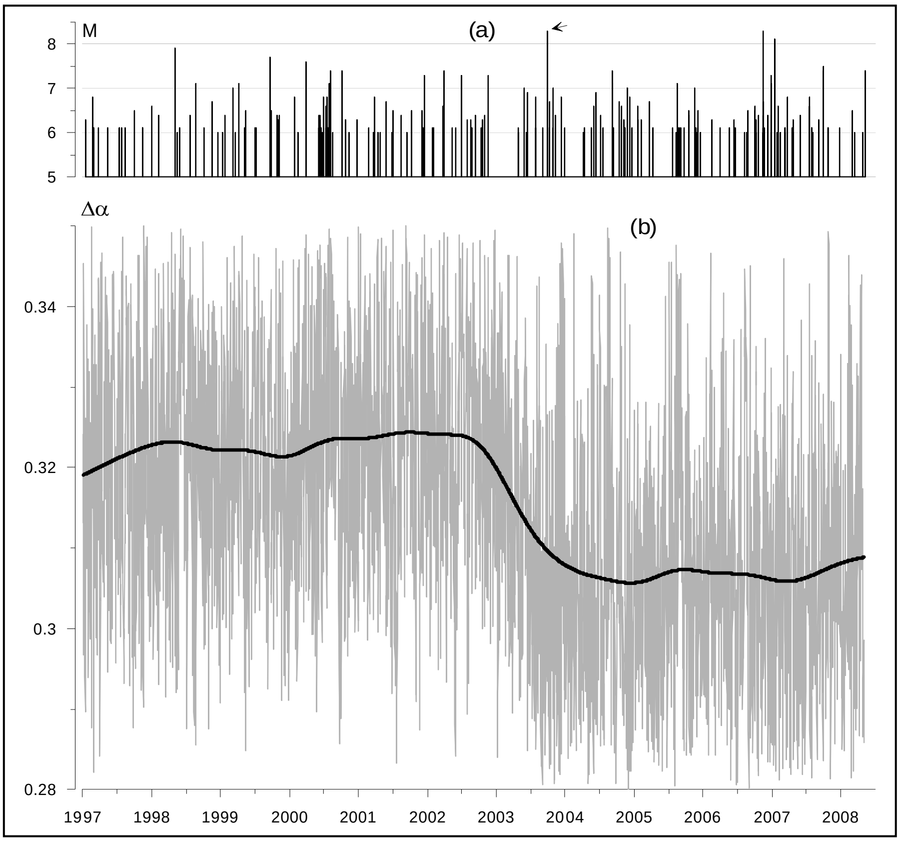


Figure 3. (a) - Sequence of strong earthquakes, arrow indicates Hokkaido earthquake; (b) - Low-frequency component (after coming to 1 day sampling) of median values of singularity spectra support width $\Delta\alpha$ (grey lines) and its mean values computed by Gaussian kernel smoothing with radius 200 days (bold black line).

Figure 4(b) presents Gaussian kernel trend with averaging radius 13 days for variations of median value of generalized Hurst exponent $\alpha^*$, estimated within 1 day time intervals after transition from 1-sec sampled signals to 1-minute sampling by averaging initial LHZ seismic records into 60 times. Thus, singularity spectra were estimated within adjacent time intervals of 1440 1-minutes samples length.

This transition from 1-sec sampling to 1 minute gives an opportunity to investigate temporal variations of multifractal properties in more low-frequency range. Such transition is not correct from a rigorous mathematical point of view because multifractal and self-similar behavior means scale invariance. But in geophysics we need investigate temporal variations of these properties within time windows of finite length. This finite length gives a limit for large scales which could be studied. Thus, in order to cover more wide range of scale it is necessary to use different sampling.

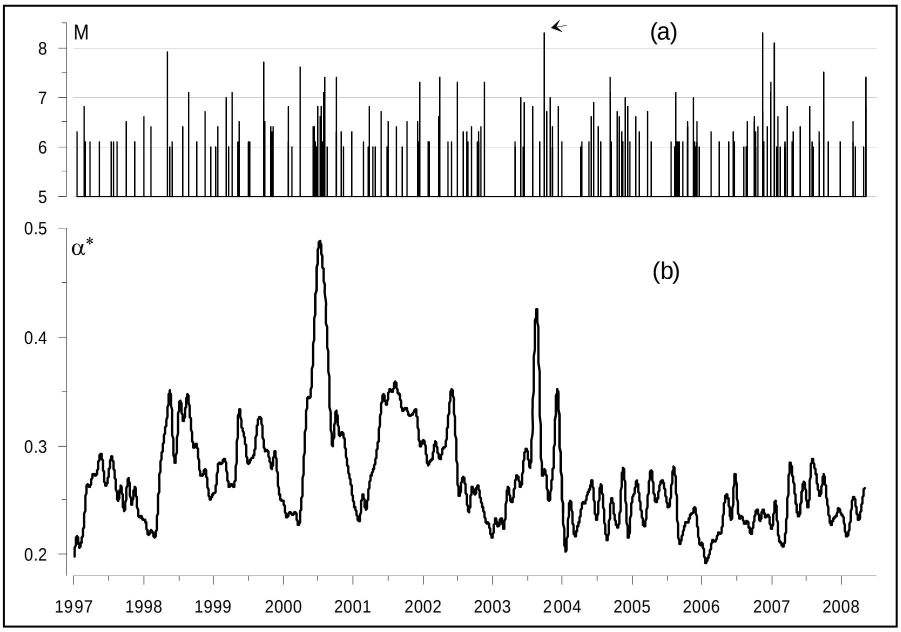


Figure 4. (a) - Sequence of strong earthquakes, arrow indicates Hokkaido earthquake; (b) - mean values (computed by Gaussian kernel smoothing with radius 13 days) of medians of singularity spectra parameter $\alpha^*$ estimated for seismic records after coming to 1-minutes sampling within adjacent time intervals of 1 day length.

The Fig.4(b) confirms a previous conclusion that Hokkaido earthquake is a change point for behavior of microseismic oscillations field at the Japan islands – it is obvious that explicit seasonal (1 year period) trend component during 1997-2003 disappear after this earthquake.

Thus, using mean (in our case – median average) of singularity spectra parameters of low-frequency microseismic noise at different stations of the net covering large seismically active region allows extracting rather thin and hidden properties of microseismic field and connect changes of these properties with seismic process.

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